Reference Sheet

Solving Equations and Systems — keep this in your binder

Name_____
Date_____
Period_____

Nothing in this unit is on the Grade 8 FAST reference sheet: the order of work for an equation, the three endings, what a solution of a system is and how to read one from a graph are all tested from memory. On this unit's test you will not have either sheet.

1. Solving an Equation

Lessons 11.01–11.04 · distribute, collect, one side, divide, check

- **Distribute.** The number outside the parentheses multiplies EVERY term inside — and a minus in front goes in with it: $15-2(x-4)=15-2x+8$.
- **Collect the like terms on each side.** Same letter, same power: $6x$ and $-x$ are like terms; $6x$ and 2 are not.
- **Take the same variable term from both sides.** Either one may go; taking the smaller keeps the number multiplying x positive.
- **Undo the number added, then divide.** What is left is a two-step equation.
- **Check in the FIRST equation, both sides:** the same number twice.

Each line	What was done
$2(3x+1)-x=2x+17$	the equation
$6x+2-x=2x+17$	distribute: 2 times 3x, 2 times 1
$5x+2=2x+17$	collect: 6x − x = 5x
$3x+2=17$	take 2x from both sides
$3x=15$	subtract 2 from both sides
$x=5$	divide both sides by 3
$2(16)-5=27$ and $2(5)+17=27$	the check: 27 on both sides

A fraction or a decimal outside the parentheses multiplies every term too.
$0.4(5x+15)=x+12$ becomes $2x+6=x+12$, so $x=6$ — not $2x+15$, the decimal to one term only.

To clear fractions, multiply EVERY term on both sides by a common denominator.
$\frac{x}{3}+2=\frac{x}{2}-1$ times 6 is $2x+12=3x-6$; take 2x from both sides, $12=x-6$; add 6, $x=18$. Check: $6+2=8$ and $9-1=8$.

2. One Solution, No Solution, Infinitely Many

Lessons 11.03–11.04 · read what is left

Simplify each side, take the same variable term from both sides, and read what is left.

What is left	The equation has	Example
x = a number	one solution	$7x-3=4x-3$: $3x=0$, so $x=0$
a = a, always true	infinitely many solutions — every number works	$5(x-1)=5x-5$: $-5=-5$
a = b, never true	no solution — no number works	$5(x-1)=5x+2$: $-5=2$

x = 0 is one solution. Zero is a number: $7x-3=4x-3$ has exactly one solution, and it is 0 — not 'no solution'.

Read the ending; do not test numbers. Five numbers that work do not show that every number works. Simplify both sides and read what is left.

The same x-terms on both sides decide it. Different x-terms: one solution. The same x-terms and the same number: infinitely many. The same x-terms and different numbers: none. $4(x+2)=4x+\square$ has infinitely many solutions with 8 in the box and none with any other number.

3. Solutions of a System

Lessons 11.05–11.06 · both equations true

A system is two equations together. Its solution is an ordered pair (x, y) that makes BOTH true.

$(5,3)$ for $y=x-2$ and $y=-x+8$: $5-2=3$ and $-5+8=3$ — a solution.

$(2,0)$: $2-2=0$ is true, but $-2+8=6$, not 0 — one equation is not enough, so it is not a solution.

x first, then y. $(3,5)$ is a different point from $(5,3)$.

Test every pair, or every system, in both equations. If the two lines cross, exactly one pair passes. If the two equations are the same line, every pair on that line passes. So never stop at the first one that works.

4. A Line from a Table of Values

Lessons 11.05–11.06 and Lesson 11.07 · x = 0 and two more

Make a table: x = 0 and two more. Put each x in and get its y. Plot the points: two draw the line and the third is the check — if it misses, a y is wrong. Every point of the line is a solution of its equation.

x	0	1	2
$y=3x-4$	−4	−1	2

A half multiplying x: use even x (for thirds, multiples of 3). Then every y is a whole number to plot.

x	0	2	4
$y=\frac{1}{2}x-1$	−1	0	1

5. How Many Solutions a System Has

Lessons 11.05–11.06 · cross, parallel, the same line

The two lines	The equations	Solutions	Example
intersect once	different coefficients of x	one: the point of intersection	$y = x - 2$, $y = -x + 8$
parallel: never meet	the same coefficient of x, different numbers added	none	$y = 3x + 1$, $y = 3x - 4$
the same line	one equation written two ways	infinitely many: every point of the line	$y = 4(x - 1)$, $y = 4x - 4$

Parallel lines have the same steepness in the same direction: the same coefficient of x. ½ and 0.5 are one number. 2 and −2 are not: $y = 2x + 1$ and $y = -2x + 1$ cross, at (0, 1). A level line is a line too: $y = 2$ and $y = -3$ have 0 multiplying x, so they are parallel, and $y = 2$ crosses $y = -x + 6$ once, at (4, 2).

The same line is infinitely many, not one and not none. Write both equations the same way first: $4(x - 1)$ is $4x - 4$.

6. Solving a System by Graphing

Lessons 11.07–11.08 · read the crossing, check it in both

Graph both lines; the solution is where they cross. Read x, then y, and check the pair in both equations. Tables find it too: the solution is the column where both tables give the same y — $y = x - 2$ and $y = -x + 8$ both give 3 at $x = 5$, so the solution is $(5, 3)$.

In a story, say what the crossing means, with its units. Club A costs $y = 8x + 15$ dollars and Club B $y = 11x$ dollars for x months. The lines cross at $(5, 55)$: at 5 months both clubs cost $55. Each line is (the amount per 1) times x, plus the start.

Between the grid lines, estimate — and say 'about'. Read each coordinate to the nearest tenth, then check the estimate in both equations: the two values come out close, not equal. A good estimate is within 0.1 of each coordinate. $y = x + 2$ and $y = -2x + 4$ cross at about $(0.7, 2.7)$: $0.7 + 2 = 2.7$ and $-2(0.7) + 4 = 2.6$ — close. (Exactly, $x = \frac{2}{3}$ and $y = \frac{8}{3}$.) The nearest grid point, (1, 3), is on $y = x + 2$ only — not a solution.

7. Vocabulary

like terms — terms with the same variable to the same power: $6x$ and $-x$

coefficient; constant — the number that multiplies the variable (6 in $6x$; the coefficient of x in $y = -x + 4$ is −1); a term with no variable

distributive property — $a(b + c) = ab + ac$ — the number outside multiplies every term inside

solution of an equation — a number that makes the equation true

no solution; infinitely many solutions — never true, whatever the number (it ends at a = b); true for every number (it ends at a = a)

system of equations — two equations about the same two variables, taken together

solution of a system — an ordered pair (x, y) that makes both equations true; on a graph, where the lines cross

ordered pair — (x, y): the x first, then the y

intersect; point of intersection — cross; the point where two lines cross — on a graph, the solution of the system

parallel lines — lines that never meet: the same coefficient of x and different numbers added

estimate — a careful read between the grid lines, written with 'about' and checked in both equations