GRADE 7 ACCELERATED · UNIT 3

Reference Sheet

Exponents and Scientific Notation — keep this in your binder

Name_____
Date_____
Period_____

Everything on this page is tested from memory. The Grade 8 FAST reference sheet does not list the laws of exponents or anything about scientific notation; what it gives you is an on-screen scientific calculator, and a calculator shows very large and very small numbers in the E form described in section 4.

1. The Laws of Exponents

Lessons 3.01–3.03, 6–7 · know all seven cold

Every law comes from writing the powers out. $2^3 \cdot 2^2 = (2 \cdot 2 \cdot 2)(2 \cdot 2) = 2^5$. If you forget a law, expand and count.

Law	Rule	Example
Product of powers	$a^m \cdot a^n = a^{m+n}$	$5^3 \cdot 5^4 = 5^7$
Quotient of powers	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{5^9}{5^4} = 5^5$
Power of a power	$(a^m)^n = a^{mn}$	$(5^3)^4 = 5^{12}$
Power of a product	$(ab)^n = a^n b^n$	$(2 \cdot 5)^3 = 2^3 \cdot 5^3$
Power of a quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{2}{5}\right)^3 = \frac{8}{125}$
Zero exponent	$a^0 = 1$ ($a \neq 0$)	$5^0 = 1$, $134^0 = 1$
Negative exponent	$a^{-n} = \frac{1}{a^n}$ ($a \neq 0$)	$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

The laws only combine LIKE bases. $2^3 \cdot 5^2$ cannot be written as one power. $2^3 \cdot 2^2$ can.

Add or subtract exponents when you multiply or divide. Multiply exponents for a power of a power. The bases never change: $2^3 \cdot 2^2$ is 2^5 , never 4^5 and never 4^6 .

Order of operations still applies. Powers before multiplication; parentheses first. $-3^2 = -9$ but $(-3)^2 = 9$. $2 \cdot 3^2 = 18$, not 36.

2. Zero and Negative Exponents

Lessons 3.02, 4, 5

The pattern that explains both. Each step to the right divides by 2:

2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

A zero exponent gives 1, not 0. Any nonzero base. $7^0 = 1$ and $(-3)^0 = 1$ and $\left(\frac{2}{9}\right)^0 = 1$.

A negative exponent means RECIPROCAL, never a negative value. 5^{-2} is $\frac{1}{25}$, a small positive number. It is not -25 and it is not -10 . Reciprocal (flip it) and opposite (change the sign) are different words.

The base keeps its own sign. $(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{-8} = -\frac{1}{8}$. The minus in the answer came from the base, not from the exponent.

A negative exponent in a denominator moves up. $\frac{1}{10^{-2}} = 10^2 = 100$.

A fraction base with a negative exponent flips. $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$.

Working backwards. $2^n = \frac{1}{32}$ means $n = -5$, because $2^5 = 32$. $10^n = 0.001$ means $n = -3$.

3. Variable Bases

Thread A, days 1–2

Same laws, letters for bases. A monomial is a coefficient times variables with exponents: $-3x^2y^5$. Multiply the coefficients; add the exponents of each variable separately.

Multiply	Raise to a power	Divide
$(4x^3y)(-2x^2y^5) = -8x^5y^6$	$(2y^2)^3 = 8y^6$	$\frac{12a^5}{4a^2} = 3a^3$
coefficients multiply; exponents add	EVERY factor is raised — the 2 as well	coefficients divide; exponents subtract

Negative exponents on variables: only the factor wearing the negative exponent moves.

$3x^{-2} = \frac{3}{x^2}$, not $\frac{1}{3x^2}$. $y^{-3}z^{-4} = \frac{1}{y^3z^4}$. $\frac{a^2}{b^{-3}} = a^2b^3$.

Finish with positive exponents. $x^4 \cdot x^{-6} = x^{-2} = \frac{1}{x^2}$. $\frac{m^3}{m^7} = \frac{1}{m^4}$.

Reciprocal pairs. x^3 and x^{-3} are reciprocals: when $x = 2$ they are 8 and $\frac{1}{8}$. Their product is $x^0 = 1$.

4. Scientific Notation

Lessons 3.08–3.09

Definition. A number written as $a \times 10^n$, where the coefficient a is **at least 1 and less than 10** and n is an integer. One nonzero digit in front of the decimal point — that is the whole rule. $26{,}000{,}000 = 2.6 \times 10^7$ and $0.00025 = 2.5 \times 10^{-4}$.

	Large number (10 or more)	Small number (less than 1)
Exponent	positive: the place of the first digit	negative: the decimal place of the first nonzero digit
Rule of thumb	count how many places the point moves LEFT to sit after the first digit	count how many places the point moves RIGHT to sit after the first nonzero digit
Example	$356{,}000 = 3.56 \times 10^5$ (five places)	$0.0000672 = 6.72 \times 10^{-5}$ (five places)
Reading back	$7.2 \times 10^4 = 72{,}000$	$3.4 \times 10^{-9} = 0.0000000034$

The exponent counts PLACES, not zeros. 0.0000672 has four zeros after the point, and the exponent is −5 because the 6 is in the fifth place. 356,000 has three zeros and the exponent is 5.

Equal in value is not the same as scientific notation. 34×10^5 equals 3,400,000, but it is not scientific notation, because 34 is not less than 10. The correct form is 3.4×10^6 .

On a calculator screen, E means × 10 to a power. 3.5E9 is $3.5\times10^{9}=3{,}500{,}000{,}000$. 4.2E−6 is $4.2\times10^{-6}=0.0000042$. E is not an error message.

Which is greater? Compare the exponents first; the larger exponent wins. Compare the coefficients only when the exponents tie. $1.2\times10^{6}>9.5\times10^{5}$. For small numbers the same rule holds: $3.9\times10^{-5}>7.4\times10^{-8}$ because −5 > −8.

5. How Many Times Larger or Smaller

Lessons 3.08–3.09

Divide the coefficients. Divide the powers of 10 (subtract the exponents). Multiply the two results.

Case	Example	Answer
Same power of 10	$\frac{6.4\times10^{7}}{3.2\times10^{7}}$	2 times
Same coefficient	$\frac{2\times10^{5}}{2\times10^{2}}$	$10^{3}=1{,}000$ times — not 3
Both different	$\frac{8.4\times10^{6}}{2.1\times10^{4}}$	$4\times10^{2}=400$ times
Negative exponents	$\frac{7.2\times10^{-4}}{1.8\times10^{-7}}$	$4\times10^{3}=4{,}000$ times; −4 − (−7) = 3
Large against small	$\frac{3\times10^{8}}{3\times10^{-4}}$	10^{12} times; 8 − (−4) = 12

A thousand, not three. 10^{3} times larger is a THOUSAND times larger. The exponent is not the answer; the power is.

'Times smaller' is the same division read the other way. If A is 4,000 times larger than B, then B is 4,000 times smaller than A.

6. Vocabulary

base — the number or variable being multiplied by itself: in 5^{3} the base is 5

exponent — the number of times the base is used as a factor: in 5^{3} the exponent is 3

power — a base with an exponent: 5^{3} is a power; its value is 125

reciprocal — the number that multiplies a given number to give 1: the reciprocal of 8 is $\frac{1}{8}$; a negative exponent asks for one

opposite — the number the same distance from 0 on the other side: the opposite of 8 is −8; a negative exponent never asks for one

monomial — a number, a variable, or a product of a number and variables with whole-number exponents: 7, x, $-3x^{2}y^{5}$

coefficient — the number factor of a monomial (-3 in $-3x^{2}y^{5}$); in scientific notation, the factor a in $a\times10^{n}$

like bases — powers of the same base: x^{2} and x^{5}; the laws combine only like bases

scientific notation — a number written as $a\times10^{n}$ with $1\le a<10$ and n an integer

standard form — a number written out with all of its digits: 26,000,000 or 0.00025

power of 10 — 10^{n}: for n positive, a 1 followed by n zeros; for n negative, a 1 in the n th decimal place