GRADE 7 ACCELERATED · UNIT 6

Reference Sheet

Relationships in Triangles — keep this in your binder

Name_____
Date_____
Period_____

Two things on this page are printed on the Grade 8 FAST reference sheet: the Triangle Inequality Theorem (section 1) and the Triangle Sum Theorem (section 5). Everything else is tested from memory — the Pythagorean Theorem and its converse are not on the state's sheet, and neither is distance on the grid, similar triangles or the angle pairs. On this unit's test you will not have either sheet. Section 5 is taught on the Thread B days, after the test; it is here because this sheet goes home with the unit.

1. The Triangle Inequality and the Converse

Lessons 6.01–6.02 · the two shorter sides; the longest as c

The Triangle Inequality: the two shorter sides must add to MORE than the longest. The FAST sheet prints it as **Triangle Inequality Theorem:** $a+b>c$; $a+c>b$; $b+c>a$ — beside a triangle whose sides are a, b and c. Adding the two shorter sides checks all three at once, because that is the only pair that can fail. It is true of every triangle, not only right ones.

Sides	The two shorter, added	Against the longest	A triangle?
3, 4, 9	3 + 4 = 7	7 < 9	No — too short to meet
4, 5, 9	4 + 5 = 9	9 = 9	No — equal lies flat
5, 6, 9	5 + 6 = 11	11 > 9	Yes
10, 4, 7	4 + 7 = 11	11 > 10	Yes — the longest is listed first

The third side is between the difference and the sum of the other two. Sides 4 and 9: the third side is more than 9 − 4 = 5 and less than 9 + 4 = 13, so $5<x<13$. Never $\leq$: a third side of exactly 5 or 13 lies flat.

The converse of the Pythagorean Theorem: square all three and test the LONGEST side as c. If the two shorter sides' squares add to the longest side's square, the triangle is right; if not, it is not — and passing the Triangle Inequality only shows there is a triangle. 24, 40, 32: $24^2+32^2=576+1{,}024=1{,}600=40^2$ — right, though the longest side is listed in the middle. 5, 6, 8: $25+36=61$, not 64 — a triangle, but not a right one.

Right triangles to know (and any whole-number multiple of one): 3-4-5, 5-12-13, 8-15-17 and 7-24-25 — so 6-8-10 and 9-12-15 as well. Anything else, square and check.

2. The Pythagorean Theorem

Lessons 6.03–6.04 · c is across from the right angle

The Pythagorean Theorem. $a^2+b^2=c^2$: the legs a and b make the right angle, and the hypotenuse c is across from it — always the longest side. It is not on the FAST sheet: memorize it.

To find	Do this	Example
the hypotenuse	square the legs, add, take the root	legs 6 and 8: 36 + 64 = 100, c = 10
a leg	square, subtract from c^2, take the root	c = 17 and a leg 15: 289 − 225 = 64, the leg is 8
a length that is not whole	the same, then the root to the nearest tenth	legs 3 and 5: 9 + 25 = 34, c = √34 ≈ 5.8

Exact, or to the nearest tenth. $\sqrt{34}$ is the exact length; 5.8 is that length to the nearest tenth, the one to give when a question asks for a measurement. Every root in this unit is of a whole number up to 225. When a question asks for the nearest tenth, the decimal is the answer — a root is never simplified here.

Check the size. A leg is shorter than the hypotenuse. A leg that comes out longer than c was found by adding the squares — a leg is found by subtracting.

In a story: find the right angle first; the side across from it is c. A ladder against a wall (the wall meets the ground square, so the ladder is c), the diagonal of a rectangle, a ramp over its run, the size of a screen (its diagonal). A 10-ft ladder with its foot 3 ft from the wall reaches $\sqrt{100-9}=\sqrt{91}\approx 9.5$ ft up the wall. Round once, at the end.

3. Distance on the Coordinate Plane

Lessons 6.05–6.06 · a slanted distance is a hypotenuse

On one line: subtract the coordinates that change — and across zero, add. From (−4, 3) to (5, 3) the y's match, so take the x's: $5-(-4)=9$ — 4 to the axis and 5 past it, never 1. Count the spaces, not the grid lines. A distance is never negative.

A slanted distance is the hypotenuse of a right triangle drawn on the grid. Draw the across leg and the up-or-down leg, read their lengths from the coordinates, then use $a^2+b^2=c^2$. From (−3, 4) to (2, −2): across 5, down 6, so $c=\sqrt{25+36}=\sqrt{61}\approx 7.8$. That is the whole method; there is no distance formula to learn.

A perimeter adds every side, and a slanted side is found by its own triangle. (1, 1), (4, 1), (4, 5): 3 + 4 + 5 = 12. When a side is a root, keep three decimal places and round only the total: 6 + 5.831 + 5.831 = 17.662 ≈ 17.7, where 6 + 5.8 + 5.8 = 17.6 is too small.

Along the streets, or straight. On a map in blocks the streets follow the grid, so the trip along them adds the legs: 6 + 8 = 14 blocks. The straight line is the hypotenuse, 10 blocks — always the shorter way, unless the two places are on one street.

4. Similar Triangles

Lessons 6.07–6.08 · one scale factor on every side

Similar triangles: matching angles are equal, and matching sides are in one ratio, the scale factor k. 6, 8, 10 and 9, 12, 15: $\frac{9}{6}=\frac{12}{8}=\frac{15}{10}=1.5$. Check all three ratios — two equal ratios are not enough. Similar sides are multiplied by one k, not added to: 3, 4, 5 with 2 added to every side is 5, 6, 7, and 5 ÷ 3, 6 ÷ 4 and 7 ÷ 5 are three different numbers.

Match the sides by size, not by where they sit on the page. Sort each triangle's sides: shortest with shortest, longest with longest. A turned or flipped copy is matched the same way, or by its equal angles.

Named triangles: the order of the letters is the matching. Triangle ABC is similar to triangle DEF, written $\triangle ABC\sim\triangle DEF$: A goes with D, B with E and C with F, so AB matches DE, BC matches EF and AC matches DF — even when the picture is turned.

A missing side: k = copy ÷ original, then multiply. A triangle 4, 6, 7 and a similar copy whose 4-side is 10: $k=\frac{10}{4}=2.5$, so its other sides are $6\times 2.5=15$ and $7\times 2.5=17.5$. A smaller copy has k less than 1.

Indirect measurement: the same sun makes the same angle. A 6-ft person casts a 4-ft shadow as a tree casts a 20-ft shadow: $k=\frac{20}{4}=5$, so the tree is $6\times 5=30$ ft. A mirror on the ground works the

same way — light leaves it at the angle it arrived — so the two right triangles are similar (the question says so), and the heights and the distances from the mirror are in one ratio.

5. Angle Facts

Thread B · after the unit test, and all year

Two angles that are	mean	Example
complementary	they add to 90°	64° and 26°
supplementary	they add to 180°	60° and 120°
vertical	across from each other where two lines cross — always equal	51° and 51°
adjacent	they share a vertex and a side, and do not overlap	a right angle split into 40° and 50°
a linear pair	adjacent and supplementary: together a straight line	130° and 50°

The Triangle Sum Theorem: the three angles of a triangle add to 180°. The FAST sheet prints it as **Triangle Sum Theorem:** $m\angle A + m\angle B + m\angle C = 180^\circ$ — beside a triangle whose corners are A, B and C; $m\angle A$ is the measure of angle A. Angles of 65° and 40°: the third angle is 180° − 105° = 75°.

The Exterior Angle Theorem: an exterior angle equals the two remote interior angles added. Remote interior angles of 50° and 70° make an exterior angle of 120° — not the interior angle beside it, which is 60° (the two make a straight line, 180°).

With x: name the fact, write its equation, solve, then put x back. A triangle's angles are x, x and 2x + 20: $x + x + (2x + 20) = 180$, so $4x = 160$ and $x = 40$ — the angles are 40°, 40° and 100°.

6. Vocabulary

Triangle Inequality Theorem — the two shorter sides of a triangle add to more than the longest side

right triangle, legs, hypotenuse — a triangle with a right angle; the two sides that make the right angle; the side across from it, always the longest

Pythagorean Theorem — in a right triangle, $a^2 + b^2 = c^2$

converse of the Pythagorean Theorem — if $a^2 + b^2 = c^2$, c the longest side, the triangle is right

Pythagorean triple — three whole numbers that are the sides of a right triangle, such as 5, 12, 13

distance — the length of the segment between two points; never negative

perimeter — the distance around a figure: every side added

similar triangles — matching angles are equal and matching sides are in one ratio

corresponding sides — a side of one triangle and the side of a similar triangle that matches it

scale factor (k) — copy ÷ original: every side of the copy is k times the side it matches

indirect measurement — finding a length you cannot reach from similar triangles you can measure

complementary, supplementary — two angles that add to 90°; two angles that add to 180°

vertical angles — the angles across from each other where two lines cross; always equal

adjacent angles — two angles that share a vertex and a side and do not overlap

exterior angle, remote interior angles — the angle between one side of a triangle and the next side extended; the two inside angles that are not beside it