GRADE 7 · UNIT 4 · LESSON 1

Deriving Area Formulas

MA.7.GR.1.1

I can build the area formulas for parallelograms, trapezoids and rhombi out of formulas I already know.

Warm-Up

1. A rectangle: base 9 cm, height 4 cm. Find its area.
2. A triangle: base 9 cm, height 4 cm. Find its area.
3. How do the two answers compare?
4. Why does the triangle formula have a one-half in it?

Warm-Up

1. A rectangle: base 9 cm, height 4 cm. Find its area. **36 cm²**
2. A triangle: base 9 cm, height 4 cm. Find its area. **18 cm²**
3. How do the two answers compare? **Exactly half.**
4. Why does the triangle formula have a one-half in it?

   Two triangles make the rectangle.

Vocabulary

quadrilateral

a polygon with exactly four sides and four vertices

parallelogram

a quadrilateral with two pairs of parallel sides

trapezoid

a quadrilateral with at least one pair of parallel sides

rhombus

a parallelogram with four sides of equal length

derive

to build a result out of something already known, rather than being told it

Notes

I. The shapes and their parts

quadrilateral — a polygon with exactly four sides and four vertices

parallelogram — a quadrilateral with two pairs of parallel sides

trapezoid — a quadrilateral with at least one pair of parallel sides

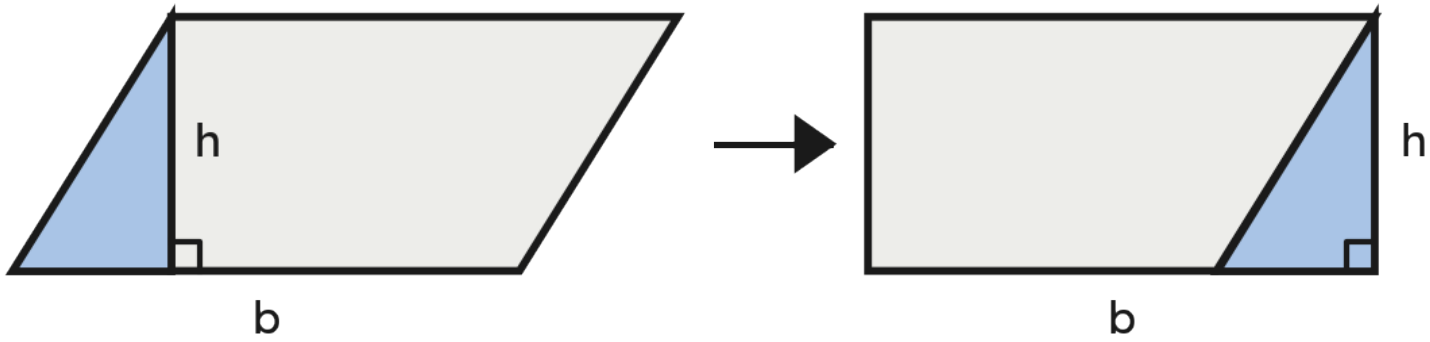
rhombus — a parallelogram with four sides of equal length

The height is always perpendicular to the base. It is never a slanted side.

Notes

II. Parallelogram: cut and slide

Cut straight down from a top vertex, perpendicular to the base. Slide the piece to the other end.



Notes

Parallelogram: the formula

Nothing was added and nothing thrown away, so the area did not change.

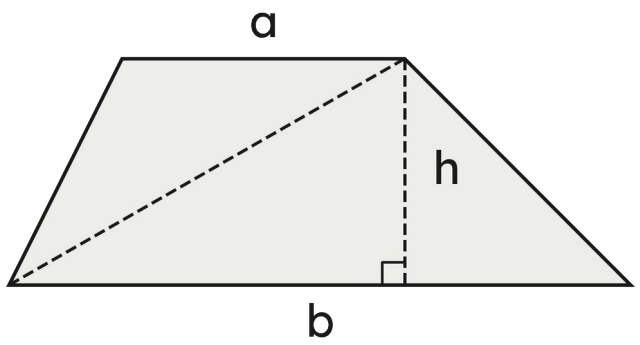
You now have a rectangle with the same base and the same height.

Area of a parallelogram = b × h — the rectangle's own formula.

Notes

III. Trapezoid: two triangles

One diagonal cuts the trapezoid into two triangles, both with the trapezoid's height h.



Notes

Trapezoid: the formula

One triangle has base a, the other base b: ½ a h + ½ b h

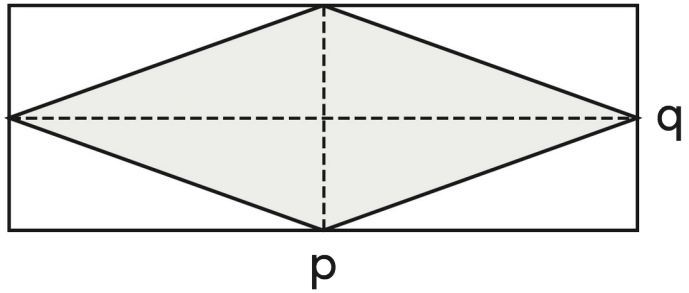
Factor out the ½ and the h: ½ h (a + b)

Area of a trapezoid = ½ h (a + b) — a and b are the two parallel sides.

Notes

IV. Rhombus: half of a rectangle

Draw both diagonals, p and q. Box the rhombus in the p × q rectangle: it fills exactly half.



Notes

Rhombus: the formula

The box is $p \times q$. The rhombus is half of it.

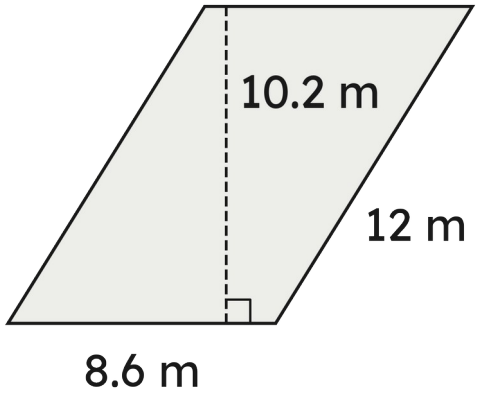
Area of a rhombus = ½ p q

Two of the three formulas have a half. Both halves come from a triangle.

Example 1

Find the area of this parallelogram.

Three numbers are printed on it.



Which is the base, which is the height, and which is neither?

Example 1

$$A = b \times h$$

$$A = 8.6 \times 10.2$$

$$A = 87.72 \text{ m}^2$$

Answer: 87.72 m²

Your Turn

A parallelogram has a base of 14 cm, a height of 6 cm, and a slanted side of 8 cm.

Find its area.

Your Turn

A parallelogram has a base of 14 cm, a height of 6 cm, and a slanted side of 8 cm.

Find its area.

$$\text{Area} = \text{base} \times \text{height}$$

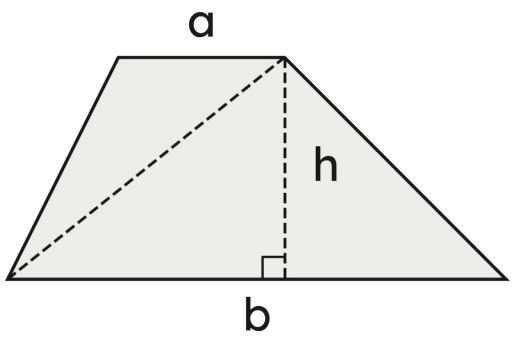
$$14 \times 6 = 84$$

Answer: 84 cm^2

Example 2

A trapezoid has parallel sides of 9 and 3 units and a height of 4 units.

One diagonal is drawn.



a = 3
b = 9
h = 4

Find the area as two triangles. Then find it with the formula.

Example 2

$$\frac{1}{2}(9)(4) + \frac{1}{2}(3)(4) = 18 + 6 = 24$$

$$\frac{1}{2}(4)(3 + 9) = \frac{1}{2}(4)(12) = 24$$

Answer: 24 units², both ways

Your Turn

A trapezoid has parallel sides 8 cm and 6 cm and height 5 cm.

Find its area both ways.

Your Turn

A trapezoid has parallel sides 8 cm and 6 cm and height 5 cm.

Find its area both ways.

Two triangles: $\frac{1}{2} \times 8 \times 5 = 20$ and $\frac{1}{2} \times 6 \times 5 = 15$

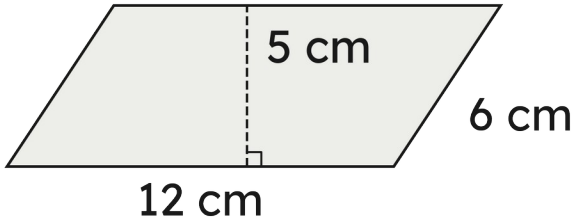
Add: $20 + 15 = 35$

Formula: $\frac{1}{2} \times 5 \times (8 + 6) = 35$

Answer: 35 cm² both ways

Whiteboards · Question 1 of 9

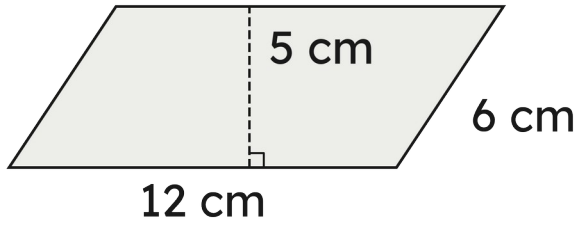
Find the area of the parallelogram.



Answer it.

Whiteboards · Question 1 of 9

Find the area of the parallelogram.



Area = base × height

$12 \times 5 = 60$

Answer: 60 cm^2

Whiteboards · Question 2 of 9

A trapezoid has bases of 8 ft and 12 ft and a height of 5 ft.

Find its area.

A. 100 ft^2

B. 30 ft^2

C. 50 ft^2

D. 240 ft^2

Answer it.

Whiteboards · Question 2 of 9

A trapezoid has bases of 8 ft and 12 ft and a height of 5 ft.

Find its area.

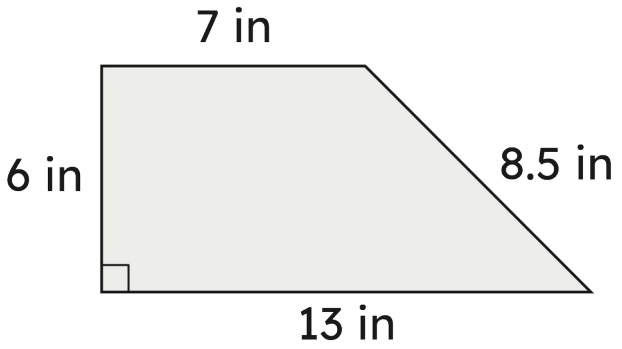
Add the bases: $8 + 12 = 20$

$$A = \frac{1}{2} \times 5 \times 20 = 50$$

Answer: C — 50 ft²

Whiteboards · Question 3 of 9

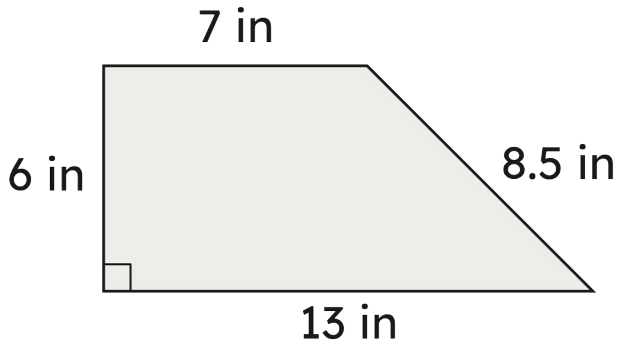
Find the area of the trapezoid.



Answer it.

Whiteboards · Question 3 of 9

Find the area of the trapezoid.



Add the bases: $7 + 13 = 20$

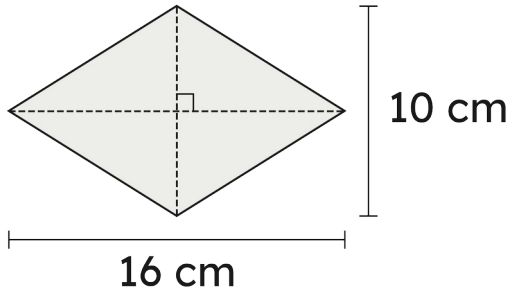
$$A = \frac{1}{2} \times 6 \times 20 = 60$$

Answer: 60 in²

Whiteboards · Question 4 of 9

A rhombus has diagonals of 10 cm and 16 cm.

Find its area.

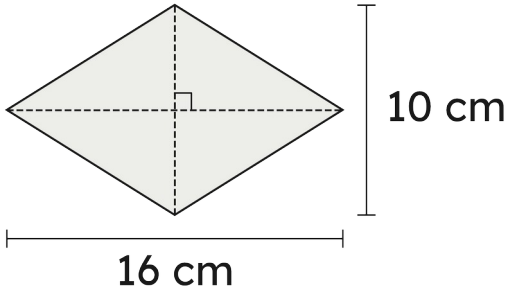


Answer it.

Whiteboards · Question 4 of 9

A rhombus has diagonals of 10 cm and 16 cm.

Find its area.



$$A = \frac{1}{2} \times 10 \times 16 = 80$$

Answer: 80 cm²

Whiteboards · Question 5 of 9

A rhombus has both diagonals 20 mm long.

Find its area.

Answer it.

Whiteboards · Question 5 of 9

A rhombus has both diagonals 20 mm long.

Find its area.

$$A = \frac{1}{2} \times 20 \times 20 = 200$$

Answer: 200 mm²

Whiteboards · Question 6 of 9

A rectangle and a parallelogram both have base 12 cm and height 5 cm.

Which has the bigger area?

Answer it.

Whiteboards · Question 6 of 9

A rectangle and a parallelogram both have base 12 cm and height 5 cm.

Which has the bigger area?

Rectangle: $12 \times 5 = 60$

Parallelogram: $12 \times 5 = 60$

Answer: Equal — 60 cm² each

Whiteboards · Question 7 of 9

A parallelogram has a base of 9 cm and a height of 4.5 cm.

Find its area.

A. 13.5 cm^2

B. 20.25 cm^2

C. 405 cm^2

D. 40.5 cm^2

Answer it.

Whiteboards · Question 7 of 9

A parallelogram has a base of 9 cm and a height of 4.5 cm.

Find its area.

$$A = 9 \times 4.5 = 40.5$$

Answer: D — 40.5 cm²

Whiteboards · Question 8 of 9

A rhombus has diagonals of 11 cm and 12 cm. Mia takes 2 minutes to cut it along the 11 cm diagonal into two triangles.

Find the height and the area of one triangle.

Answer it.

Whiteboards · Question 8 of 9

A rhombus has diagonals of 11 cm and 12 cm. Mia takes 2 minutes to cut it along the 11 cm diagonal into two triangles.

Find the height and the area of one triangle.

Height: $12 \div 2 = 6$

$$A = \frac{1}{2} \times 11 \times 6 = 33$$

Answer: $h = 6$ cm, $A = 33$ cm²

Whiteboards · Question 9 of 9

A trapezoid has bases 5 in and 11 in and height 6 in.

Cut it into two triangles of height 6. Find each area, add them, then check with the formula. Write all three steps.

Write your answer in sentences.

Whiteboards · Question 9 of 9

A trapezoid has bases 5 in and 11 in and height 6 in.

Cut it into two triangles of height 6. Find each area, add them, then check with the formula. Write all three steps.

Triangles: $\frac{1}{2} \times 11 \times 6 = 33$ and $\frac{1}{2} \times 5 \times 6 = 15$

Add: $33 + 15 = 48$

Formula: $\frac{1}{2} \times 6 \times (5 + 11) = 48$

Answer: 48 in², both ways

Before You Go

Every formula today came out of a rectangle or a triangle.

The height is perpendicular. The slanted side is never the height.

Tomorrow nothing is derived — everything is used.

IXL

1. Open IXL and start today's skills.
2. Work on paper where the question needs work. The answer box does not show it.
3. Every skill listed is required, to a SmartScore of 60. Due by the Unit 4 test.

Today's skills — all required

Area of a parallelogram (P6T)
Area of a trapezoid (JYV)
Area of rhombuses (DPB)