GRADE 7 · UNIT 4 · LESSON 3

Finding the Area of Polygons

MA.7.GR.1.2

I can find the area of a regular polygon by cutting it into shapes I already have formulas for.

Warm-Up

1. A regular hexagon, each side 5 cm. Find its perimeter.
2. A triangle: base 5 cm, height 4.3 cm. Find its area.
3. Six of question 2's triangles make a shape. Its area?
4. Why does adding the six areas give the whole area?

Warm-Up

1. A regular hexagon, each side 5 cm. Find its perimeter.

   30 cm

2. A triangle: base 5 cm, height 4.3 cm. Find its area.

   10.75 cm^2

3. Six of question 2's triangles make a shape. Its area?

   64.5 cm^2

4. Why does adding the six areas give the whole area?

   Nothing left out, nothing counted twice.

Vocabulary

regular polygon

a polygon with all sides equal and all angles equal

apothem

the perpendicular distance from the center to a side

radius (of a polygon)

the distance from the center to a vertex — longer than the apothem, and not used in the area

decompose

to cut a figure into shapes you already know how to measure

Notes

I. Regular polygons and their parts

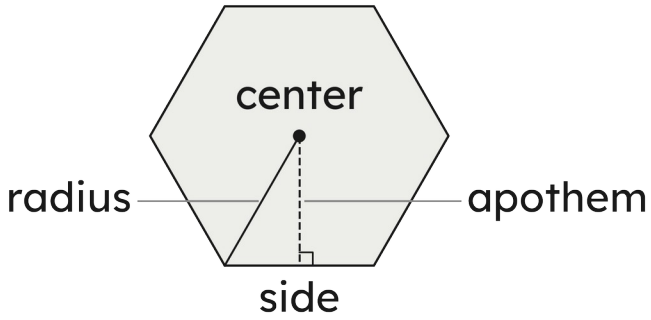
regular polygon — a polygon with all sides equal and all angles equal

apothem — the perpendicular distance from the center to a side

Notes

The apothem and the radius

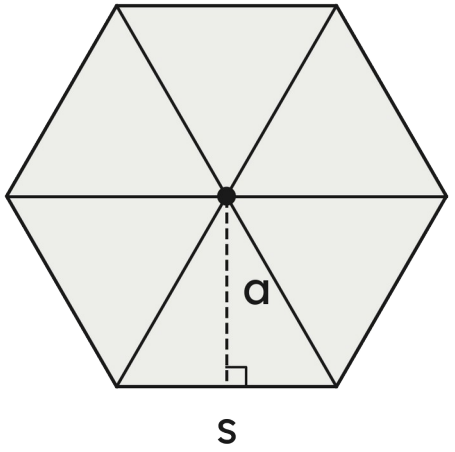
The radius goes from the center to a vertex, so it is longer. It is almost never what you need.



Notes

II. Cut it into congruent triangles

Draw a segment from the center to every vertex.



Notes

Congruent triangles: the formula

An n-sided regular polygon gives n triangles, and they are all the same.

Each triangle has base s — a side — and height a — the apothem.

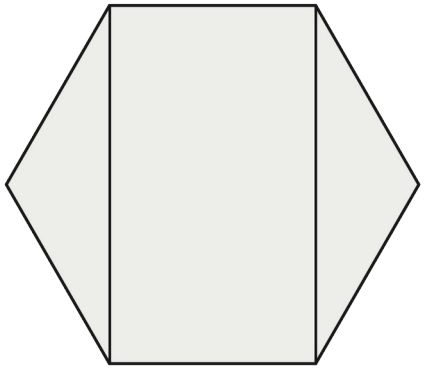
One triangle: $\frac{1}{2} s a$. The whole polygon: $n \times \frac{1}{2} s a$.

Notes

III. Cut it into triangles and quadrilaterals

You do not have to cut from the center.

Straight cuts can give a rectangle in the middle and a piece on each side.



Notes

Pieces that are not congruent

Now the pieces are not congruent, so you must find each area and add.

Any correct cutting gives the same total. Pick the one whose lengths you were given.

Notes

IV. Rounding and units

Do not round in the middle. Round once, at the end.

Rounding one triangle's area and then multiplying by nine multiplies the error by nine.

An apothem printed on a figure is usually already rounded.

So two correct methods can differ in the last digit. That is not an error.

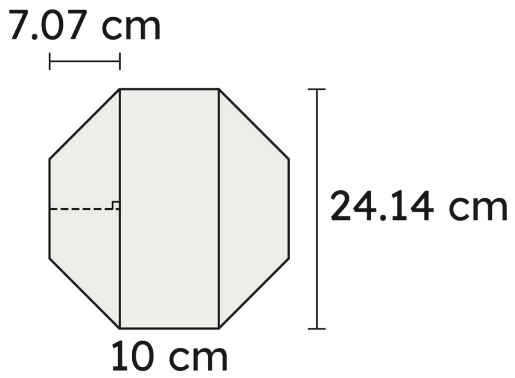
Area units are always squared.

Example 1

Find the area of a regular octagon: sides 10 cm, apothem 12.07 cm.

Cut it from the center: how many triangles?

The picture cuts it with two straight lines instead: 24.14 cm high, 7.07 cm at each end. What pieces?



Which cut uses the side and the apothem?

Example 1

$$8 \times \frac{1}{2}(10)(12.07) = 482.8$$

$$10(24.14) + 2 \cdot \frac{1}{2}(7.07)(24.14 + 10)$$

$$= 241.4 + 241.4 = 482.8 \text{ cm}^2$$

Answer: 482.8 cm² either way

Your Turn

A regular pentagon has sides of 6 cm and an apothem of 4.13 cm.

Find its area.

Your Turn

A regular pentagon has sides of 6 cm and an apothem of 4.13 cm.

Find its area.

One triangle: $\frac{1}{2} \times 6 \times 4.13 = 12.39$

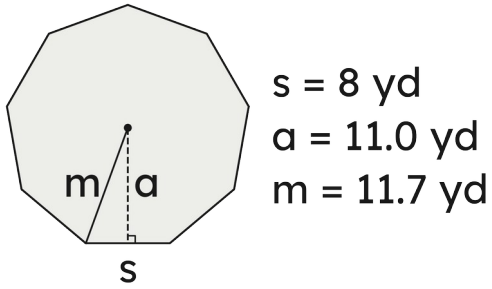
Five triangles: $5 \times 12.39 = 61.95$

Answer: 61.95 cm²

Example 2

Vernon found the area of a regular nonagon. He made two mistakes.

$$A = \frac{1}{2}(8)(11.7) = 46.8 \text{ yd}^2 \qquad A = 18(46.8) = 842.4 \text{ yd}^2$$



s = 8 yd
a = 11.0 yd
m = 11.7 yd

Find both mistakes.

Example 2

He used m — the radius — where the apothem belongs.

A nonagon has 9 triangles, not 18.

$$9 \times \frac{1}{2}(8)(11.0) = 9 \times 44 = 396 \text{ yd}^2$$

Answer: 396 yd², not 842.4

Your Turn

A regular decagon has sides of 4 cm, an apothem of 6.2 cm, and 6.5 cm from the center to a vertex.

Which length is not needed? Find the area.

Your Turn

A regular decagon has sides of 4 cm, an apothem of 6.2 cm, and 6.5 cm from the center to a vertex.

Which length is not needed? Find the area.

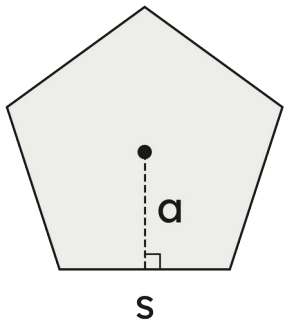
One triangle: $\frac{1}{2} \times 4 \times 6.2 = 12.4$

Ten triangles: $10 \times 12.4 = 124$

Answer: The 6.5 cm is not needed; 124 cm²

Whiteboards · Question 1 of 9

Find the area of the regular pentagon.

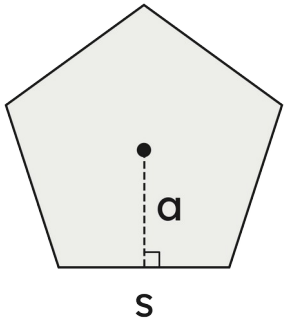


s = 12 in
a = 8.3 in

Answer it.

Whiteboards · Question 1 of 9

Find the area of the regular pentagon.



s = 12 in
a = 8.3 in

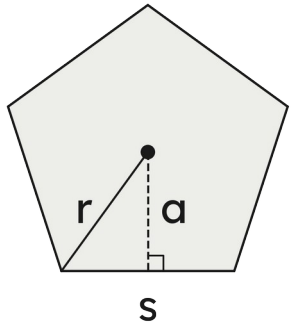
One triangle: $\frac{1}{2} \times 12 \times 8.3 = 49.8$

Five triangles: $5 \times 49.8 = 249$

Answer: 249 in²

Whiteboards · Question 2 of 9

Find the area of the regular pentagon.

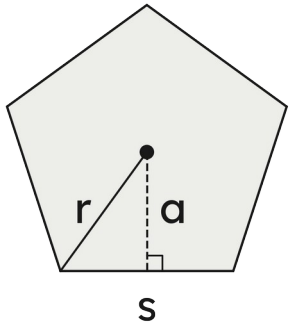


s = 12 in
a = 8.3 in
r = 10.2 in

Answer it.

Whiteboards · Question 2 of 9

Find the area of the regular pentagon.



s = 12 in
a = 8.3 in
r = 10.2 in

Use the apothem, 8.3. The radius is not needed.

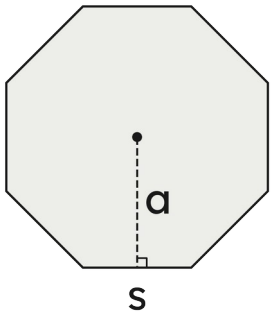
One triangle: $\frac{1}{2} \times 12 \times 8.3 = 49.8$

Five triangles: $5 \times 49.8 = 249$

Answer: 249 in² — same as question 1

Whiteboards · Question 3 of 9

Find the area of the regular octagon.

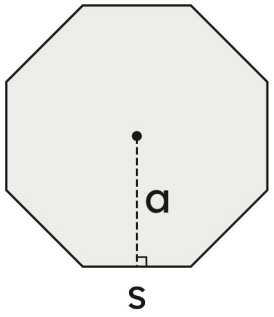


s = 6 m
a = 7.2 m

Answer it.

Whiteboards · Question 3 of 9

Find the area of the regular octagon.



s = 6 m
a = 7.2 m

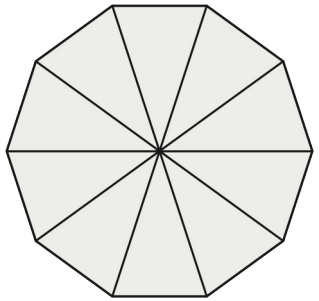
One triangle: $\frac{1}{2} \times 6 \times 7.2 = 21.6$

Eight triangles: $8 \times 21.6 = 172.8$

Answer: 172.8 m²

Whiteboards · Question 4 of 9

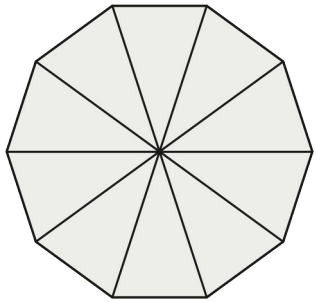
Cut from the center into congruent triangles. How many are there?



Answer it.

Whiteboards · Question 4 of 9

Cut from the center into congruent triangles. How many are there?



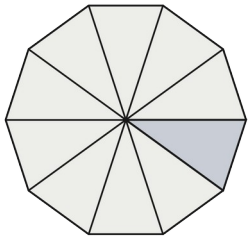
One triangle for each side. A decagon has 10 sides.

Answer: 10 triangles

Whiteboards · Question 5 of 9

One triangle is 8.5 cm².

Find the area of the decagon.



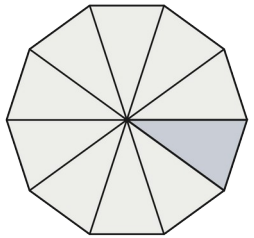
one triangle = 8.5 cm²

Answer it.

Whiteboards · Question 5 of 9

One triangle is 8.5 cm².

Find the area of the decagon.



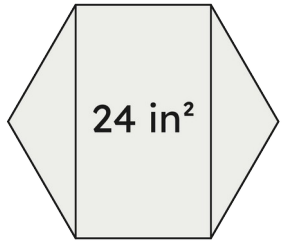
one triangle = 8.5 cm²

Ten triangles: $10 \times 8.5 = 85$

Answer: 85 cm²

Whiteboards · Question 6 of 9

Cut into a rectangle and two triangles. Find the hexagon's area.

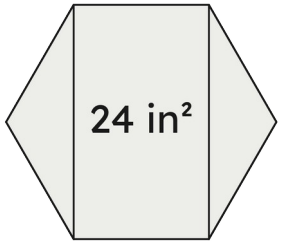


each end triangle: 6 in²

Answer it.

Whiteboards · Question 6 of 9

Cut into a rectangle and two triangles. Find the hexagon's area.



each end triangle: 6 in²

The pieces are not the same size, so add.

$24 + 6 + 6 = 36$

Answer: 36 in²

Whiteboards · Question 7 of 9

A regular hexagon has sides of 7 ½ in and an apothem of 6.5 in.

Find its area, to the nearest square inch.

A. 146 in^2

B. 147 in^2

C. 292.5 in^2

D. 24 in^2

Answer it.

Whiteboards · Question 7 of 9

A regular hexagon has sides of 7 ½ in and an apothem of 6.5 in.

Find its area, to the nearest square inch.

One triangle: $\frac{1}{2} \times 7.5 \times 6.5 = 24.375$

Six triangles: $6 \times 24.375 = 146.25$

To the nearest square inch: 146

Answer: A — 146 in²

Whiteboards · Question 8 of 9

A regular decagon has sides of 9.3 in, an apothem of 14.3 in, and 15.0 in from the center to a vertex.

Find its area, to the nearest square inch.

A. 664.95 in^2

B. 66.5 in^2

C. 697.5 in^2

D. 665 in^2

Answer it.

Whiteboards · Question 8 of 9

A regular decagon has sides of 9.3 in, an apothem of 14.3 in, and 15.0 in from the center to a vertex.

Find its area, to the nearest square inch.

One triangle: $\frac{1}{2} \times 9.3 \times 14.3 = 66.495$

Ten triangles: $10 \times 66.495 = 664.95$

To the nearest square inch: 665

Answer: D — 665 in²

Whiteboards · Question 9 of 9

A regular hexagon cuts into a rectangle 10 m by 17.3 m and two triangles, each with base 17.3 m and height 5 m.

Find the hexagon's area, to the nearest square meter.

Write your answer in sentences.

Whiteboards · Question 9 of 9

A regular hexagon cuts into a rectangle 10 m by 17.3 m and two triangles, each with base 17.3 m and height 5 m.

Find the hexagon's area, to the nearest square meter.

Rectangle: $10 \times 17.3 = 173$

Each triangle: $\frac{1}{2} \times 17.3 \times 5 = 43.25$

Add: $173 + 43.25 + 43.25 = 259.5$

To the nearest square meter: 260

Answer: 173 + 43.25 + 43.25 = 259.5, so 260 m²

Before You Go

There is no formula for a nine-sided shape. There does not need to be.

Cut it into shapes you already have formulas for, then add.

Multiply only when the pieces are congruent. Round once, at the end.

IXL

1. Open IXL and start today's skills.
2. Work on paper where the question needs work. The answer box does not show it.
3. Every skill listed is required, to a SmartScore of 60. Due by the unit test.

Today's skills — all required

Area of a parallelogram (P6T)
Area of a trapezoid (JYV)
Area of rhombuses (DPB)