GRADE 7 · UNIT 5 · LESSON 4

Making Population Predictions — Part 2

MA.7.DP.1.3

I can find the size of a population from a sample, including by tagging and recatching.

Warm-Up

1. 24% of a random sample said yes. Predict for 900.
2. 14 reds in 40 spins. Predict reds in 300 spins.
3. Find the median: 12, 7, 9, 15, 4, 10
4. Write $\frac{21}{180}$ in lowest terms.

Warm-Up

1. 24% of a random sample said yes. Predict for 900. **216**
2. 14 reds in 40 spins. Predict reds in 300 spins. **105**
3. Find the median: 12, 7, 9, 15, 4, 10 **9.5**
4. Write $\frac{21}{180}$ in lowest terms. $\frac{7}{60}$

Vocabulary

capture–recapture

tag some, put them back, catch a new sample, and use the tagged fraction to size the population

population size

the number you are solving for when the sample is what you know

Notes

I. The population can be the missing number

A random sample of 50 club members: 4 walk to meetings. The sign-in sheet counts 24 walkers in all.

$$\frac{4}{50} = \frac{2}{25} \qquad 24 = 2 \cdot 12 \qquad \frac{2}{25} = \frac{24}{300}$$

Scale the sample's fraction up until its top is the count you know. The bottom is the population.

About 300 members. Check it forwards: 2/25 of 300 is 24.

Notes

II. Tag, put back, catch again

Tag 30 alligators and put them back. Later catch 150 at random: 12 are tagged.

$$\frac{12}{150} = \frac{2}{25} \qquad 30 = 2 \cdot 15 \qquad \frac{2}{25} = \frac{30}{375}$$

The tagged fraction in the catch is the tagged fraction in the lake.

About 375 alligators in the lake.

Notes

III. What has to be true

The second catch is random — tagged and untagged equally catchable.

Nothing joins or leaves between the two catches.

The answer is an estimate, like every prediction in this unit.

Example 1

A ranger tags 45 geese and lets them go.

Later she catches 160 at random; 18 of them are tagged.

About how many geese are in the park?

Example 1

$$\frac{18}{160} = \frac{9}{80}$$

$$45 = 9 \cdot 5$$

$$\frac{9}{80} = \frac{45}{400}$$

Answer: About 400 geese

Your Turn

A biologist tags 48 turtles and lets them go.

Later she catches 200 at random; 16 are tagged. About how many turtles are in the pond?

Your Turn

A biologist tags 48 turtles and lets them go.

Later she catches 200 at random; 16 are tagged. About how many turtles are in the pond?

Tagged fraction of the catch: $\frac{16}{200} = \frac{2}{25}$

$48 = 2 \cdot 24$

$\frac{2}{25} = \frac{48}{600}$, so the whole is $25 \cdot 24 = 600$

Answer: About 600 turtles

Example 2

In a random sample of 60 students at a school, 9 have a library card.

The library has given cards to 42 of the school's students.

About how many students are in the school?

Example 2

$$\frac{9}{60} = \frac{3}{20}$$

$$42 = 3 \cdot 14$$

$$\frac{3}{20} = \frac{42}{280}$$

Answer: About 280 students

Your Turn

In a random sample of 50 of a town's drivers, 8 use the toll road.

The toll road lists 72 of the town's drivers. About how many drivers does the town have?

Your Turn

In a random sample of 50 of a town's drivers, 8 use the toll road.

The toll road lists 72 of the town's drivers. About how many drivers does the town have?

$$\frac{8}{50} = \frac{4}{25}$$

$$72 = 4 \cdot 18$$

$\frac{4}{25} = \frac{72}{450}$, so the whole is $25 \cdot 18 = 450$

Answer: About 450 drivers

Whiteboards · Question 1 of 9

A ranger tags 81 fish and returns them. Later she catches 200 at random; 18 are tagged.

Estimate the number of fish in the lake.

Answer it.

Whiteboards · Question 1 of 9

A ranger tags 81 fish and returns them. Later she catches 200 at random; 18 are tagged.

Estimate the number of fish in the lake.

Tagged fraction of the catch: $\frac{18}{200} = \frac{9}{100}$

$81 = 9 \cdot 9$

$\frac{9}{100} = \frac{81}{900}$, so the whole is $100 \cdot 9 = 900$

Answer: About 900

Whiteboards · Question 2 of 9

A random sample of 50 students found 3 who bike to school, and 33 students bike in all.

About how many students are in the school?

Answer it.

Whiteboards · Question 2 of 9

A random sample of 50 students found 3 who bike to school, and 33 students bike in all.

About how many students are in the school?

$\frac{3}{50}$ of the sample bike

$33 = 3 \cdot 11$

$\frac{3}{50} = \frac{33}{550}$, so the whole is $50 \cdot 11 = 550$

Answer: About 550

Whiteboards · Question 3 of 9

A ranger tags 50 birds. Later she catches 300 at random, and 25 are tagged.

Which proportion estimates the number of birds, n?

A. $\frac{25}{300} = \frac{n}{50}$

B. $\frac{50}{300} = \frac{25}{n}$

C. $n = 50 \cdot 300$

D. $\frac{25}{300} = \frac{50}{n}$

Answer it.

Whiteboards · Question 3 of 9

A ranger tags 50 birds. Later she catches 300 at random, and 25 are tagged.

Which proportion estimates the number of birds, n?

Tagged, out of caught: $\frac{25}{300} = \frac{1}{12}$

Tagged, out of all the birds: $\frac{50}{n}$

$\frac{1}{12} = \frac{50}{600}$, so $n = 600$

Answer: D — tagged over all, on both sides

Whiteboards · Question 4 of 9

A ranger tags 60 turtles and returns them. Later she catches 180 at random; 24 are tagged.

Estimate the number of turtles.

Answer it.

Whiteboards · Question 4 of 9

A ranger tags 60 turtles and returns them. Later she catches 180 at random; 24 are tagged.

Estimate the number of turtles.

Tagged fraction of the catch: $\frac{24}{180} = \frac{2}{15}$

$60 = 2 \cdot 30$

$\frac{2}{15} = \frac{60}{450}$, so the whole is $15 \cdot 30 = 450$

Answer: About 450

Whiteboards · Question 5 of 9

A random sample of 90 of a store's 1,400 shoppers found 27 who used a coupon.

Predict the number of shoppers who use a coupon.

Answer it.

Whiteboards · Question 5 of 9

A random sample of 90 of a store's 1,400 shoppers found 27 who used a coupon.

Predict the number of shoppers who use a coupon.

$$\frac{27}{90} = \frac{3}{10}$$

$$\frac{3}{10} \cdot 1{,}400 = 420$$

Answer: About 420

Whiteboards · Question 6 of 9

A random sample of 30 hens on a farm: 7 lay brown eggs.

Which asks for the size of the population?

A. 600 hens in all: how many lay brown?

B. What fraction of the 30 lay brown?

C. 49 lay brown: how many hens in all?

D. How many of the 30 do not lay brown?

Answer it.

Whiteboards · Question 6 of 9

A random sample of 30 hens on a farm: 7 lay brown eggs.

Which asks for the size of the population?

In C the number of hens on the farm is missing.

$\frac{7}{30} = \frac{49}{210}$: $30 \cdot 7 = 210$

Answer: C — the farm's number of hens is missing

Whiteboards · Question 7 of 9

Over 4 days a ranger tags 35 deer and returns them. Later she catches 160 at random; 28 are tagged.

Estimate the number of deer.

Answer it.

Whiteboards · Question 7 of 9

Over 4 days a ranger tags 35 deer and returns them. Later she catches 160 at random; 28 are tagged.

Estimate the number of deer.

Tagged fraction of the catch: $\frac{28}{160} = \frac{7}{40}$

$35 = 7 \cdot 5$

$\frac{7}{40} = \frac{35}{200}$, so the whole is $40 \cdot 5 = 200$

Answer: About 200

Whiteboards · Question 8 of 9

A random sample of 25 of a county's voters found 3 who signed a petition, and 84 signed in all.

About how many voters are in the county?

Answer it.

Whiteboards · Question 8 of 9

A random sample of 25 of a county's voters found 3 who signed a petition, and 84 signed in all.

About how many voters are in the county?

$\frac{3}{25}$ of the sample signed

$84 = 3 \cdot 28$

$\frac{3}{25} = \frac{84}{700}$, so the whole is $25 \cdot 28 = 700$

Answer: About 700

Whiteboards · Question 9 of 9

A ranger tags lizards on an island, then catches a random sample.

Why does her catch tell her about the whole island?

Write your answer in sentences.

Whiteboards · Question 9 of 9

A ranger tags lizards on an island, then catches a random sample.

Why does her catch tell her about the whole island?

A random catch has about the island's tagged fraction.

Answer: Her catch is random, so it stands for the island.

Independent Practice

1. A ranger tags 25 fish and returns them. Later she catches 120 at random; 15 are tagged. Estimate the lake's fish.

2. A random sample of 100 students found 11 who play an instrument, and 44 play in all. About how many students are in the school?

3. A random sample of 64 tablets: 16 had a dead pixel. Predict the faulty tablets in 960.

4. Over 5 nights a biologist tags 50 bats and returns them. Later 140 are caught at random and 20 are tagged. Estimate the colony.

5. A student says the lake in question 1 holds 3,000 fish because $25 \cdot 120 = 3{,}000$. In one sentence, say what went wrong.

6. A random sample of 100 of a town's households found 9 that compost, and the town's records show 63 that compost. How many households are there, and what is the forward check?

Before You Go

The tagged fraction in the catch is the tagged fraction in the lake.

Scale the sample up until the top is the count you know.

Next: scale factors — the same proportion, on a drawing.

IXL

1. Open IXL and start today's skills.
2. Work on paper where the question needs work. The answer box does not show it.
3. Every skill listed is required, to a SmartScore of 60. Due by the Unit 5 test.

Today's skills — all required

Make an estimate from sample data (FMC)

Use collected data to make predictions (EXS)